



Enjoy solving complex problems using creative mathematical thinking? So do we!

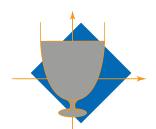
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And of course - Good Luck!

Voliš rješavati teške probleme koristeći kreativno matematičko razmišljanje? I mi!

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I naravno - Puno sreće!



14TH EUROPEAN MATHEMATICAL CUP



 13^{th} December 2025 - 21^{st} December 2025 Senior Category

Problem 1. Let $k \ge 2$ be an integer. Let m and n be coprime positive integers with exactly k positive divisors such that m < n.

For $i \in \{1, ..., k\}$, denote by f_i and d_i the *i*-th smallest divisor of m and n, respectively. Suppose that

$$d_i - f_i \mid n - m$$

for all $i \in \{2, ..., k\}$. Prove that $d_i \geqslant f_i$ for all $i \leqslant \frac{k}{2}$.

(Ivan Novak)

Problem 2. Let $f: \mathbb{N} \to \mathbb{N}$ (where \mathbb{N} is a set of positive integers) be a function such that for every positive integer k, the set $\{f(1), f(2), \ldots, f(k)\}$ contains exactly f(f(k)) elements. Prove that

$$f(f(f(k))) = f(k)$$

for every positive integer k.

(Ivan Novak)

Problem 3. Let ABC be an acute triangle with circumcircle ω . Let the angle bisector of $\angle B$ intersect AC, ω , and the parallel to AB from C in D, E and F respectively. Let X be the intersection of ω and the circumcircle of triangle DCF, and let Y be a point on CF such that YF = YD. The line XF intersects ω and DY in T and P respectively. The circumcircle of triangle $\triangle TDE$ meets the lines PF and EY in EX and EX. Prove that the circumcircles of triangles EX0 and EX2 are internally tangent.

(Yasser Merabet)

Problem 4. Determine all sequences of positive real numbers a_1, a_2, a_3, \ldots , such that for each positive integer n the following equality holds:

$$a_n + \max(a_{n+1}, a_{n+2}) = \frac{1}{\min(a_n, a_{n+1})}.$$

(Ivan Novak)

Time: 240 minutes.

Each problem is worth 10 points.