14TH EUROPEAN MATHEMATICAL CUP

13th December 2025 - 21st December 2025

Junior Category

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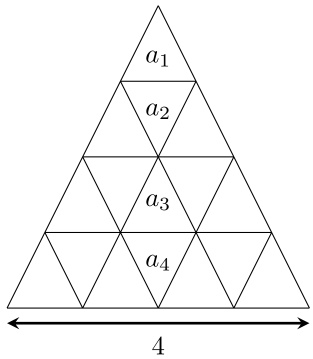
Problem 1. Let ω_1 and ω_2 be two circles intersecting at A and B. The common tangent, closer to A, of ω_1 and ω_2 touches ω_1 at P and ω_2 at Q. The tangent of ω_1 at A meets ω_2 at C, which is different from A, and the extension of AP meets QC at D. Let E be the center of circumcircle of triangle ABD. The lines AD and QE intersect at F. Prove that F lies on the circle with diameter PQ.

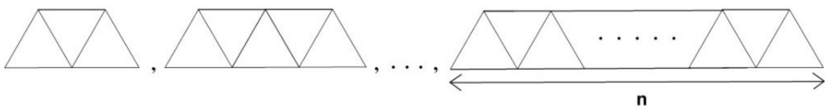
(Steve Vo Dinh)

Problem 2. Let n be a positive integer. Divide an equilateral triangle of side length n into equilateral triangles of side length one. Here is an example shown below for $n = 4$:



Label the small equilateral triangles through which one of the altitudes of the large equilateral triangle passes as $a_1, a_2, \dots, a_n$ (see illustration above how to do in in case of $n = 4$).

Let $f(i)$ denote the number of ways to tile the large equilateral triangle using exactly one of each



such that the triangle a_i is removed.

- If n is even, determine $f(2) + f(4) + \cdots + f(2k) + \cdots + f(n)$.
- Prove that $f(1) + f(2) + \cdots + f(n) \geqslant 2^{n-2}$, for all positive integers n.

(Karlo Jokoš)

Problem 3. Determine the largest positive integer n for which there exist positive integers a and q such that

$$q^6 \leq n \quad \text{and} \quad \left|\sqrt{2} - \frac{a}{q}\right| \leq \frac{1}{\sqrt{n}}.$$

(Miroslav Marinov)

Problem 4. Find all positive integers n with the following property:
For every positive integer d which divides n, there exists a positive integer k which divides n such that

$$d + n \mid dn + k.$$

(Ivan Novak)

Time: 240 minutes. *Each problem is worth 10 points.*

The use of calculators or any other instruments except rulers and compasses is not permitted. Cheating in any form is strictly prohibited. Sharing or posting the problems or solutions before the end of the competition (22nd of December) to anyone who might participate is strictly prohibited.