13TH European Mathematical Cup

MLADI NADARENI MATEMATIČARI **Marin Getaldić**

14^{th} December 2024 - 22^{nd} December 2024

Junior Category

Problem 1. Wiske wrote a 2024-digit positive integer on the blackboard. In each round of the game she erases the last digit of the integer, let this digit be d, and writes down the sum of the remaining number and $2d$ in place of the old number. She repeats the same steps with the newly obtained number. After a certain number of rounds, Wiske found that the new number obtained was the same as the number in the last round and she stopped the game. What is the smallest possible 2024-digit integer that Wiske started with in this game?

(Kai Chen)

Problem 2. Let X be the largest possible value of the expression

$$\min\{bc, 2-a^2\} + \min\{ac, 2-b^2\} + \min\{ab, 2-c^2\},$$

where a, b and c are positive real numbers. Similarly, let Y be the smallest possible value of the expression

$$\max\{a^2, 2-bc\} + \max\{b^2, 2-ac\} + \max\{c^2, 2-ab\},$$

where a, b and c are positive real numbers. Prove that $X = Y$. *(Ognjen Tešić)*

Problem 3. Let ABC be a triangle with incenter I and incircle ω. Let ℓ be the tangent to ω parallel to BC and distinct from BC. Let D be the intersection of ℓ and AC, and let M be the midpoint of $\overline{ID}$. Prove that $\angle AMD = \angle DBC$.

(Weihua Wang)

Problem 4. Let $\mathcal{F}$ be a family of (distinct) subsets of the set $\{1, 2, \ldots, n\}$ such that for all $A, B \in \mathcal{F}$ we have that $A^c \cup B \in \mathcal{F}$, where A^c is the set of all members of $\{1, 2, \ldots, n\}$ that are not in A.

Prove that every $k \in \{1, 2, \ldots, n\}$ appears in at least half of the sets in $\mathcal{F}$.

(Stijn Cambie, Mohammad Javad Moghaddas Mehr)

Time: 240 minutes. *Each problem is worth 10 points.*

The use of calculators or any other instruments except rulers and compasses is not permitted. Cheating in any form is strictly prohibited. Sharing or posting the problems or solutions before the end of the competition (22nd of December) to anyone who might participate is strictly prohibited.