European Mathematical Cup

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I naravno - **Puno sreće!**

12^{TH} European Mathematical Cup

9^{th} December 2023 - 17^{th} December 2023

Senior Category

Problem 1. Determine all sets of real numbers S such that:

- 1 is the smallest element of S,
- for all $x, y \in S$ such that $x > y$, $\sqrt{x^2 - y^2} \in S$.

(Adian Anibal Santos Sepčić)

Problem 2. Let ABC be a triangle such that $\angle BAC = 90°$. The incircle of triangle ABC is tangent to the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ at D, E, F respectively. Let M be the midpoint of $\overline{EF}$. Let P be the projection of A onto BC and let K be the intersection of MP and AD. Prove that the circumcircles of triangles AFE and PDK have equal radius.

(Kyprianos-Iason Prodromidis)

Problem 3. Let n be a positive integer. Let B_n be the set of all binary strings of length n. For a binary string $s_1 s_2 \ldots s_n$, we define its twist in the following way. First, we count how many blocks of consecutive digits it has. Denote this number by b. Then, we replace s_b with $1 - s_b$. A string a is said to be a *descendant* of b if a can be obtained from b through a finite number of twists. A subset of B_n is called *divided* if no two of its members have a common descendant. Find the largest possible cardinality of a divided subset of B_n.

Remark. Here is an example of a twist: $10110 \to 10100$ because $1|0|11|0$ has 4 blocks of consecutive digits.

(Viktor Simjanoski)

Problem 4. Let $f : \mathbb{N} \to \mathbb{N}$ be a function such that for all positive integers x and y, the number $f(x) + y$ is a perfect square if and only if $x + f(y)$ is a perfect square. Prove that f is injective.

Remark. A function $f : \mathbb{N} \to \mathbb{N}$ is injective if for all pairs (x, y) of distinct positive integers, $f(x) \neq f(y)$ holds.

(Ivan Novak)

Time: 240 minutes. *Each problem is worth 10 points.*

The use of calculators or any other instruments except rulers and compasses is not permitted. Cheating in any form is strictly prohibited. Sharing or posting the problems or solutions before the end of the competition (18th of December) to anyone who might participate is strictly prohibited.