7^{TH} European Mathematical Cup

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Senior Category

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Problem 1. A partition of a positive integer is *even* if all its elements are even numbers. Similarly, a partition is *odd* if all its elements are odd. Determine all positive integers n such that the number of even partitions of n is equal to the number of odd partitions of n.

Remark: A *partition* of a positive integer n is a non-decreasing sequence of positive integers whose sum of elements equals n. For example, $(2, 3, 4)$, $(1, 2, 2, 2, 2)$ and (9) are partitions of 9.

(Ivan Novak)

Problem 2. Let ABC be a triangle with $|AB| < |AC|$. Let k be the circumcircle of $\triangle ABC$ and let O be the center of k. Point M is the midpoint of the arc $\widehat{BC}$ of k not containing A. Let D be the second intersection of the perpendicular line from M to AB with k and E be the second intersection of the perpendicular line from M to AC with k. Points X and Y are the intersections of CD and BE with OM respectively. Denote by k_b and k_c circumcircles of triangles BDX and CEY respectively. Let G and H be the second intersections of k_b and k_c with AB and AC respectively. Denote by k_a the circumcircle of triangle AGH.

Prove that O is the circumcenter of $\triangle O_aO_bO_c$, where O_a, O_b, O_c are the centers of k_a, k_b, k_c respectively.

(Petar Nizić-Nikolac)

Problem 3. For which real numbers $k > 1$ does there exist a bounded set of positive real numbers S with at least 3 elements such that

$$k(a - b) \in S$$

for all $a, b \in S$ with $a > b$?

Remark: A set of positive real numbers S is *bounded* if there exists a positive real number M such that $x < M$ for all $x \in S$.

(Petar Nizić-Nikolac)

Problem 4. Let x, y, m, n be integers greater than 1 such that

$$\underbrace{x^{x^{\cdot^{\cdot^{x}}}}}_{m \text{ times}} = \underbrace{y^{y^{\cdot^{\cdot^{y}}}}}_{n \text{ times}}.$$

Does it follow that $m = n$?

Remark: This is a tetration operation, so we can also write ${}^{m}x = {}^{n}y$ for the initial condition.

(Petar Nizić-Nikolac)