



7TH EUROPEAN MATHEMATICAL CUP
8th December 2018 - 16th December 2018
Junior Category



Problem 1. Let a, b, c be non-zero real numbers such that

$$a^2 + b + c = \frac{1}{a},$$

$$b^2 + c + a = \frac{1}{b},$$

$$c^2 + a + b = \frac{1}{c}.$$

Prove that at least two of a, b, c are equal.

(Daniel Paleka)

Problem 2. Find all pairs (x, y) of positive integers such that

$$xy \mid x^2 + 2y - 1.$$

(Ivan Novak)

Problem 3. Let ABC be an acute triangle with $|AB| < |AC|$ and orthocenter H . The circle with center A and radius $|AC|$ intersects the circumcircle of $\triangle ABC$ at point D and the circle with center A and radius $|AB|$ intersects the segment \overline{AD} at point K . The line through K parallel to CD intersects BC at the point L . If M is the midpoint of \overline{BC} and N is the foot of the perpendicular from H to AL , prove that the line MN bisects the segment \overline{AH} .

(Miroslav Marinov)

Problem 4. Let n be a positive integer. Ana and Banana are playing the following game:

First, Ana arranges $2n$ cups in a row on a table, each facing upside-down. She then places a ball under a cup and makes a hole in the table under some other cup. Banana then gives a finite sequence of commands to Ana, where each command consists of swapping two adjacent cups in the row.

Her goal is to achieve that the ball has fallen into the hole during the game. Assuming Banana has no information about the position of the hole and the position of the ball at any point, what is the smallest number of commands she has to give in order to achieve her goal?

(Adrian Beker)