6TH European Mathematical Cup

9th December 2017–17th December 2017

Junior Category

Problem 1. Find all pairs (x, y) of integers that satisfy the equation

$$x^2y + y^2 = x^3.$$

(Daniel Paleka)

Problem 2. A regular hexagon in the plane is called *sweet* if its area is equal to 1. Is it possible to place 2000000 sweet hexagons in the plane such that the union of their interiors is a convex polygon of area at least 1900000?

Remark: A subset S of the plane is called *convex* if for every pair of points in S, every point on the straight line segment that joins the pair of points also belongs to S. The hexagons may overlap.

(Josip Pupić, Borna Vukorepa)

Problem 3. Let ABC be an acute triangle. Denote by H and M the orthocenter of ABC and the midpoint of side BC, respectively. Let Y be a point on AC such that YH is perpendicular to MH and let Q be a point on BH such that QA is perpendicular to AM. Let J be the second point of intersection of MQ and the circle with diameter MY. Prove that HJ is perpendicular to AM.

(Steve Dinh)

Problem 4. The real numbers x, y, z satisfy $x^2 + y^2 + z^2 = 3$. Prove the inequality

$$x^3 - (y^2 + yz + z^2)x + y^2z + yz^2 \leq 3\sqrt{3}$$

and find all triples (x, y, z) for which equality holds.

(Miroslav Marinov)