4^{TH} European Mathematical Cup

5^{th} December 2015–13^{th} December 2015

Senior Category

Mladi nadareni matematičari **Marin Getaldić**

Problem 1. $A = \{a, b, c\}$ is a set containing three positive integers. Prove that we can find a set $B \subset A$, $B = \{x, y\}$ such that for all odd positive integers m, n we have

$$10 | x^m y^n - x^n y^m.$$

(Tomi Dimovski)

Problem 2. Let a, b, c be positive real numbers such that $abc = 1$. Prove that

$$\frac{a+b+c+3}{4} \geqslant \frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}.$$

(Dimitar Trenevski)

Problem 3. Circles k_1 and k_2 intersect in points A and B, such that k_1 passes through the center O of the circle k_2. The line p intersects k_1 in points K and O and k_2 in points L and M, such that the point L is between K and O. The point P is orthogonal projection of the point L to the line AB. Prove that the line KP is parallel to the M-median of the triangle ABM. *(Matko Ljulj)*

Problem 4. A group of mathematicians is attending a conference. We say that a mathematician is k-content if he is in a room with at least k people he admires or if he is admired by at least k other people in the room. It is known that when all participants are in a same room then they are all at least $3k + 1$-content. Prove that you can assign everyone into one of 2 rooms in a way that everyone is at least k-content in his room and neither room is empty. *Admiration is not necessarily mutual and no one admires himself.*

(Matija Bucić)

Time allowed: 240 minutes.

Each problem is worth 10 points.

Calculators are not allowed.