



2ND EUROPEAN MATHEMATICAL CUP
7th December 2013–15th December 2013
Senior Category



Problem 1. In each field of a table there is a real number. We call such $n \times n$ table *silly* if each entry equals the product of all the numbers in the neighbouring fields.

- Find all 2×2 silly tables.
- Find all 3×3 silly tables.

(Two fields of a table are neighbouring if they share a common side.)

(Borna Vukorepa)

Problem 2. *Palindrome* is a sequence of digits which doesn't change if we reverse the order of its digits. Prove that a sequence $(x_n)_{n=0}^{\infty}$ defined as

$$x_n = 2013 + 317n$$

contains infinitely many numbers with their decimal expansions being palindromes.

(Stijn Cambie)

Problem 3. We call a sequence of n digits one or zero a *code*. Subsequence of a code is a *palindrome* if it is the same after we reverse the order of its digits. A palindrome is called *nice* if its digits occur consecutively in the code. (Code (1101) contains 10 palindromes, of which 6 are nice.)

- What is the least number of palindromes in a code?
- What is the least number of nice palindromes in a code?

(Ognjen Stipetić)

Problem 4. Given a triangle ABC let D, E, F be orthogonal projections from A, B, C to the opposite sides respectively. Let X, Y, Z denote midpoints of AD, BE, CF respectively. Prove that perpendiculars from D to YZ , from E to XZ and from F to XY are concurrent.

(Matija Bucić)

Time allowed: 240 minutes.

Each problem is worth 10 points.

Calculators are not allowed.