

1st BALKAN STUDENT MATHEMATICAL COMPETITION

1. Matematičko natjecanje učenika Balkana

November 2008.

2nd grade

Problem 1. If x , y and z are positive real numbers for which $x + y + z = 1$, prove the inequality

$$\frac{1}{\sqrt{x+y}} + \frac{1}{\sqrt{y+z}} + \frac{1}{\sqrt{z+x}} \leq \frac{1}{\sqrt{2xyz}}.$$

(Adrian Satja Kurdija)

Problem 2. A natural number is written in each cell of 10×10 table. It is known that, no matter which 5 columns and 5 rows of this table we choose, the sum of numbers in their 25 intersection cells is even. Prove that all the numbers in the table are even.

Problem 3. Let M_n and N_n be points on sides \overline{CA} and \overline{CB} of triangle ABC , respectively, such that

$$|CM_n| = \frac{1}{n} |CA|, \quad |CN_n| = \frac{1}{n+1} |CB|, \quad \forall n \in \mathbb{N}.$$

Find the locus of points $M_i N_i \cap M_j N_j$, where i and j are different natural number.

Note. $M_i N_i \cap M_j N_j$ denotes the intersection of lines $M_i N_i$ and $M_j N_j$.

Problem 4. a , b and c are natural numbers. It is known that $a^2 + b^2 + abc$ has no more than 2008 natural divisors and that it is divisible by $(c+2)^{1004}$. Prove that a and b are not relatively prime.

(Adrian Satja Kurdija)

Time allowed: 240 minutes.

Each problem is worth 10 points.

Write each problem on a separate paper.

Calculators or any other helping items, excluding rulers and compasses, are not allowed.